1. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of $(1 + ax)^7$, where a is a constant. Give each term in its simplest form.

(4)

Given that the coefficient of x^2 in this expansion is 525,

(b) find the possible values of a.

(2)
(Total 6 marks)

2. Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(3 - x)^6$$

and simplify each term.

(Total 4 marks)

3. (a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(2 + kx)^7$$

where k is a constant. Give each term in its simplest form.

(4)

Given that the coefficient of x^2 is 6 times the coefficient of x,

(b) find the value of k.

(2)
(Total 6 marks)

4. Find the first 3 terms, in ascending powers of x, of the binomial expansion of $(3 - 2x)^5$, giving each term in its simplest form.

(Total 4 marks)

5. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of $(1 + ax)^{10}$, where a is a non-zero constant. Give each term in its simplest form.

(4)

Given that, in this expansion, the coefficient of x^3 is double the coefficient of x^3,

(b) find the value of a.

(2)
(Total 6 marks)

6. (a) Find the first 4 terms of the expansion of $\left(1+\frac{x}{2}\right)^{10}$ in ascending powers of x, giving each term in its simplest form.

(4)

(b) Use your expansion to estimate the value of $(1.005)^{10}$, giving your answer to 5 decimal places.

(3)
(Total 7 marks)

7. (a) Find the first four terms, in ascending powers of x, in the binomial expansion of $(1 + kx)^6$,where k is a non-zero constant.

(3)

Given that, in this expansion, the coefficients of x and x^2 are equal, find

(b) the value of k,

(2)

(c) the coefficient of x^3.

(1)
(Total 6 marks)

8. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of $(1 - 2x)^5$. Give each term in its simplest form.

(4)

(b) If x is small, so that x^2 and higher powers can be ignored, show that

$$(1 + x)(1 - 2x)^5 \approx 1 - 9x.$$

(2)
(Total 6 marks)

9. Find the first 3 terms, in ascending powers of x, of the binomial expansion of $(2 + x)^6$, giving each term in its simplest form.

(Total 4 marks)

10. (a) Write down the binomial expansion, in ascending powers of x, of $(1 + 6x)^4$, giving each coefficient as an integer.

(3)

(b) Use your binomial expansion to find the exact value of 601^4.

(2)
(Total 5 marks)

11. (a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(1 + px)^9,$$

where p is a constant.

(2)

These first 3 terms are 1, $36x$ and qx^2, where q is a constant.

(b) Find the value of p and the value of q.

(4)
(Total 6 marks)

12. (a) Write down the first three terms, in ascending powers of x, of the binomial expansion of $(1 + px)^{12}$, where p is a non-zero constant.

(2)

Given that, in the expansion of $(1 + px)^{12}$, the coefficient of x is $(-q)$ and the coefficient of x^2 is $11q$,

(b) find the value of p and the value of q.

(4)
(Total 6 marks)

13. In the binomial expansion, in ascending powers of x, of $(1 + ax)^n$, where a and n are constants, the coefficient of x is 15. The coefficient of x^2 and of x^3 are equal.

(a) Find the value of a and the value of n.

(6)

(b) Find the coefficient of x^3.

(1)
(Total 7 marks)

14. Find the first three terms, in ascending powers of x, of the binomial expansion of $(3 + 2x)^5$, giving each term in its simplest form.

(Total 4 marks)

15. (a) Find the first four terms, in ascending powers of x, in the binomial expansion of $\left(k + \frac{x}{2}\right)^5$, where k is a constant.

(2)

Given that the third term of this series is $540x^2$,

(b) show that $k = 6$,

(2)

(c) find the coefficient of x^3.

(2)
(Total 6 marks)

16. For the binomial expansion, in descending powers of x, of

$$\left(x^3 - \frac{1}{2x}\right)^{12},$$

(a) find the first 4 terms, simplifying each term.

(5)

(b) Find, in its simplest form, the term independent of x in this expansion.

(3)
(Total 8 marks)

17. (a) Write down the first 4 terms of the binomial expansion, in ascending powers of x, of $(1 + ax)^n$, $n > 2$.

(2)

Given that, in this expansion, the coefficient of x is 8 and the coefficient of x^2 is 30,

(b) calculate the value of n and the value of a,

(4)

(c) find the coefficient of x^3.

(2)
(Total 8 marks)

18. The expansion of $(2 - px)^6$ in ascending powers of x, as far as the term in x^2, is

$$64 + Ax + 135x^2.$$

Given that $p > 0$, find the value of p and the value of A.

(Total 7 marks)

(c) find the coefficient of x^3.

(2)
(Total 8 marks)

19. The first three terms in the expansion, in ascending powers of x, of $(1 + px)^n$, are $1 - 18x + 36p^2x^2$. Given that n is a positive integer, find the value of n and the value of p.

(Total 7 marks)

20. (a) Expand $(2 + \frac{1}{4}x)^9$ in ascending powers of x as far as the term in x^3, simplifying each term.

(4)

(b) Use your series, together with a suitable value of x, to calculate an estimate of $(2.025)^9$.

(2)
(Total 6 marks)

21. The first four terms, in ascending powers of x, of the binomial expansion of $(1 + kx)^n$ are

$$1 + Ax + Bx^2 + Bx^3 + \ldots,$$

where k is a positive constant and A, B and n are positive integers.

(a) By considering the coefficients of x^2 and x^3, show that $3 = (n - 2)\,k$.

(4)

Given that $A = 4$,

(b) find the value of n and the value of k.

(4)

(Total 8 marks)

21. (a) $(1+ax)^7 = 1+7ax....$ or $1+7(ax)....$ (<u>Not</u> unsimplified versions) B1

$+\dfrac{7\times6}{2}(ax)^2 + \dfrac{7\times6\times5}{6}(ax)^3$ Evidence from <u>one</u> of these terms is enough

$+21a^2x^2$ or $+21(ax)^2$ or $+21(a^2x^2)$ A1

$+35a^3x^3$ or $+35(ax)^3$ or $+35(a^3x^3)$ A1 4

<u>Note</u>

The terms can be 'listed' rather than added.

Requires correct structure: a correct binomial coefficient in any form (perhaps from Pascal's triangle) with the correct power of x
Allow missing a's and wrong powers of a, e.g.

$$\frac{7\times6}{2}ax^2, \qquad \frac{7\times6\times5}{3\times2}x^3$$

However, $21 + a^2x^2 + 35 + a^3x^3$ or similar is M0.

$1+ 7ax + 21+ a^2x^2 + 35 + a^3x^3 = 57 + ...$ scores the B1 (isw).

$\begin{pmatrix}7\\2\end{pmatrix}$ and $\begin{pmatrix}7\\3\end{pmatrix}$ or equivalent such as ${}^7C_2$ and ${}^7C_3$ are acceptable, but

<u>not</u> $\left(\dfrac{7}{2}\right)$ or $\left(\dfrac{7}{3}\right)$ (unless subsequently corrected).

1st A1: Correct x^2 term. 2nd A1: Correct x^3 term (The binomial coefficients <u>must</u> be simplified).

<u>Special case</u>:
If $(ax)^2$ and $(ax)^3$ are seen within the working, but then lost…
… A1 A0 can be given if $21ax^2$ and $35ax^3$ are <u>both</u> achieved.

<u>a's omitted throughout:</u>
Note that only the M mark is available in this case.

(b) $21a^2 = 525$

$a = \pm5$ (Both values are required) A1
(The answer $a = 5$ with no working scores A0) 2

<u>Note</u>

M: Equating their coefficent of x^2 to 525.

An equation in a or a^2 alone is required for this M mark, but allow 'recovery' that shows <u>the required coefficient</u>, e.g.

$21a^2x^2 = 525 \Rightarrow 21a^2 = 525$ is acceptable,

but $21a^2x^2 = 525 \Rightarrow a^2 = 25$ is not acceptable.

After $21ax^2$ in the answer for (a), allow 'recovery' of a^2 in (b) so that full marks are available for (b) (but not retrospectively for (a)).

[6]

2. $\left[(3-x)^6 =\right] 3^6 + 3^5 \times 6 \times (-x) + 3^4 \times \binom{6}{2} \times (-x)^2$

$= 729, \quad -1458x, \quad +1215x^2$ B1 A1 A1

–

Note

for <u>either</u> the x term <u>or</u> the x^2 term. Requires <u>correct</u> binomial coefficient in any form with the correct power of x – condone lack of negative sign and wrong power of 3. This mark may be given if no working is shown, but one of the terms including x is correct.

Allow $\frac{6}{1}$, or $\frac{6}{2}$ (must have a power of 3, even if only power 1)

First term must be 729 for **B1**, (writing just 3^6 is **B0**) can isw if numbers added to this constant later. Can allow 729(1…
Term must be simplified to –1458x for **A1cao**. The x is required for this mark. **Final A1**is c.a.o and needs to be $+1215x^2$ (can follow omission of negative sign in working)

Descending powers of x would be $x^6 + 3 \times 6 \times (-x)^5 + 3^2 \times \binom{6}{4} \times (-x)^4 + \ldots$

i.e. $x^6 - 18x^5 + 135x^4 + ..$ This is M1B1A0A0 if completely "correct"
or B0A0A0 for <u>correct</u> binomial coefficient in any form with the correct power of x as before

–

Alternative

NB Alternative method: $(3-x)^6 = 3^6(1 + 6 \times (-\frac{x}{3}) + \binom{6}{2} \times (-\frac{x}{3})^2 + \ldots)$

is **M1B0A0A0** – answers must be simplified to 729, $-1458x$, $+1215x^2$ for full marks (awarded as before)

The mistake $(3-x)^6 = 3(1-\frac{x}{3})^6 = 3(1 + 6 \times (-\frac{x}{3}) + \times \binom{6}{2} \times (-\frac{x}{3})^2 + ..)$

may also be awarded **M1B0A0A0**

Another mistake $3^6(1 - 6x + 15x^2 ...) = 729...$ would be M1B1A0A0

[4]

3. (a) $(7 \times ... \times x)$ or $(21 \times ... \times x^2)$ The 7 or 21 can be in 'unsimplified' form.

$(2 + kx)^7 = 2^7 + 2^6 \times 7 \times kx + 2^5 \times \binom{7}{2} k^2 x^2$

$= 128; + 448kx, +672k^2x^2$ [or $672(kx)^2$] B1; A1, A1 4

(If $672kx^2$ follows $672(kx)^2$, isw and allow A1)

Note

The terms can be 'listed' rather than added. Ignore any extra terms.

for either the x term or the x^2 term. Requires correct binomial coefficient in any form with the correct power of x, but the other part of the coefficient (perhaps including powers of 2 and/or k) may be wrong or missing.

Allow binomial coefficients such as $\binom{7}{1}$, $\binom{7}{1}$, $\binom{7}{2}$, ${}^7C_1$, ${}^7C_2$.

However, $448 + kx$ or similar is M0.

B1, A1, A1 for the simplified versions seen above.

Alternative:

Note that a factor 2^7 can be taken out first: $2^7\left(1+\frac{kx}{2}\right)^7$, but the mark scheme still applies.

Ignoring subsequent working (isw):

Isw if necessary after correct working:

e.g. $128 + 448kx + 672k^2 x^2$ B1 A1 A1

$= 4 + 14kx + 21k^2 x^2$ isw

(Full marks are still available in part (b)).

(b) $6 \times 448k = 672k^2$

$k = 4$ (Ignore $k = 0$, if seen) A1 2

Note

for equating their coefficient of x^2 to 6 times that of x… to get an equation in k, … or equating their coefficient of x to 6 times that of x^2, to get an equation in k.

Allow this M mark even if the equation is trivial, providing their coefficients from part (a) have been used, e.g. $6 \times 448k = 672k$, but beware $k = 4$ following from this, which is A0.

An equation in k alone is required for this M mark, so...

e.g. $6 \times 448kx = 672k^2 x^2 \Rightarrow k = 4$ or similar is M0 A0 (equation in coefficients only is never seen), but ...

e.g. $6 \times 448kx = 672k^2 x^2 \Rightarrow 6 \times 448k = 672k^2 \Rightarrow k = 4$ will get A1

(as coefficients rather than terms have now been considered).

The mistake $2\left(1+\frac{kx}{2}\right)^7$ would give a maximum of 3 marks:

M1B0A0A0, M1A1

[6]

4. $(3-2x)+ = 243$, …… $+ 5 \times (3)^4 (-2x) = -810x$ …… B1 B1

$+\frac{5\times 4}{2}(3)^3(-2x)^2 =$ $+ 1080x^2$ A1 4

__Notes__

First term must be 243 for **B1**, writing just 3^5 is B0 (Mark their final answers except in second line of special cases below).
Term must be simplified to $-810x$ for **B1**
The x is required for this mark.

The **method** mark (**is generous** and is awarded for an attempt at Binomial to get the third term.
There must be an x^2 (or no x– i.e. not wrong power) and attempt at Binomial Coefficient and at dealing with powers of 3 and 2. The power of 3 should not be one, but the power of 2 may be one (regarded as bracketing slip).
So allow $\binom{5}{2}$ or $\binom{5}{3}$ $^5C_2$ or $^5C_3$ or even $\binom{5}{2}$ or $\binom{5}{3}$ or use of '10' (maybe from Pascal's triangle)

May see $^5C_3\,(3)^3\,(-2x)^2$ or $^5C_2\,(3)^3\,(-2x^2)$ or $^5C_2\,(3)^5\,(-\frac{2}{3}x^2)$ or $10(3)^3(2x)^2$

which would each score the

A1is c.a.o and needs $1080x^2$ (if $1080x^2$ is written with no working this is awarded both marks i.e. **A1.)**

Special cases

$243 + 810x + 1080x^2$ is **B1B0M1A1** (condone no negative signs)
Follows correct answer with $27 - 90x + 120x^2$ can isw here (sp case)– full marks for correct answer
Misreads ascending and gives $-32x^5 + 240x^4 - 720x^3$ is marked as **B1B0M1A0** special case and must be completely correct. *(If any slips could get B0B0M1A0)*
Ignores 3 and expands $(1 \pm 2x)^5$ is **0/4**
$243, -810x, 1080x^2$ is full marks but 243, –810, 1080 is **B1,B0,** **A0**

NB Alternative method $3^5\,(1-\frac{2}{3}x)^5 = 3^5 - 5 \times 3^5 \times (\frac{2}{3}x) +$

$\binom{5}{3}3^5\,(-\frac{2}{3}x)^2 +$.. is **B0B0M1A0** – answers must be simplified to 243

$-810x + 1080x^2$ for full marks (awarded as before)

Special case $3(1-\frac{2}{3}x)^5 = 3 - 5 \times 3 \times (\frac{2}{3}x) + \binom{5}{3}3(-\frac{2}{3}x)^2 +$.. is

B0, B0, **A0**

Or $3(1-2x)^5$ is **B0B0M0A0**

[4]

5. (a) $(1 + ax)^{10} = 1 + 10ax$....... (<u>Not</u> unsimplified versions) B1

$+\frac{10\times 9}{2}(ax)^2 + \frac{10\times 9\times 8}{6}(ax)^3$ Evidence from <u>one</u> of these terms is sufficient

$+ 45(ax)^2$, $+ 120(ax)^3$ or $+ 45a^2x^2$, $+ 120a^3x^3$ A1, A1 4

The terms can be 'listed' rather than added.

Requires correct structure: 'binomial coefficient' (perhaps from Pascal's triangle) and the correct power of x.
(The M mark can also be given for an expansion in <u>descending</u> powers of x).
Allow 'slips' such as:
$\frac{10\times 9}{2}ax^2, \frac{10\times 9}{3\times 2}(ax)^3, \frac{10\times 9}{2}x^2, \frac{9\times 8\times 7}{3\times 2}a^3x^3$
However, $45 + a^2x^2 + 120 + a^3x^3$ or similar is M0.
$\binom{10}{2}$ and $\binom{10}{3}$ or equivalent such as ${}^{10}C_2$ and ${}^{10}C_3$
are acceptable, and even $\left(\frac{10}{2}\right)$ and $\left(\frac{10}{3}\right)$ are acceptable
for the method mark.

1st A1: Correct x^2 term.

2nd A1: Correct x^3 term (These <u>must</u> be simplified).
If simplification is not seen in (a), but correct simplified terms are seen in (b), these marks can be awarded. However, if <u>wrong</u> simplification is seen in (a), this takes precedence.

<u>Special case:</u>
If $(ax)^2$ and $(ax)^3$ are seen within the working, but then lost…
… A1 A0 can be given if $45ax^2$ and $120ax^3$ are <u>both</u> achieved.

(b) $120a^3 = 2 \times 45a^2$ $a = \frac{3}{4}$ or equiv. $\left(\text{e.g. } \frac{90}{120}, 0.75\right)$ Ignore $a = 0$, if seen A1 2

M: Equating their coefficent of x^3 to twice their coefficient of x^2… <u>or</u> equating their coefficent of x^2 to twice their coefficient of x^3. (... or coefficients can be <u>correct</u> coefficients rather than their coefficients).
Allow this mark even if the equation is trivial, e.g. $120a = 90a$.
An equation in a alone is required for this M mark, although condone, e.g. $120a^3x^3 = 90a^2x^2 \Rightarrow (120a^3 = 90a^2 \Rightarrow)$
$a = \frac{3}{4}$.

<u>Beware</u>: $a = \frac{3}{4}$ following $120a = 90a$, which is A0.

[6]

6. (a) $\left(1+\frac{1}{2}x\right)^{10} = \underline{1+\binom{10}{1}\left(\frac{1}{2}x\right)+\binom{10}{2}\left(\frac{1}{2}x\right)^2+\binom{10}{3}\left(\frac{1}{2}x\right)^3}$ M1A1

$= 1 + 5x; \ +\frac{45}{4}$ (or 11.25)$x^2 + 15x^3$

(coeffs need to be these, i.e, simplified) A1; A1 4

[Allow A1 A0, if totally correct with unsimplified, single fraction coefficients)

For first A1: Consider underlined expression only.
Requires correct structure for at least two of the three terms:

(i) Must be attempt at binomial coefficients.
[Be generous :allow all notations e.g. ${}^{10}C_2$, even $\left(\frac{10}{2}\right)$; allow "slips".]

(ii) Must have increasing powers of x,

(iii) May be listed, need not be added; *this applies for all marks*.

First A1: Requires **all three** correct terms but need not be simplified, allow 1^{10} etc, ${}^{10}C_2$ etc, and condone omission of brackets around powers of ½ x

Second A1: Consider as B1: **1 + 5*x***

(b) $(1 + \frac{1}{2} \times 0.01)^{10} = 1 + 5(0.01) + (\frac{45}{4}$ or $11.25)\,(0.01)^2 + 15(0.01)^3$ M1A1ft
$= 1 + 0.05 + 0.001125 + 0.000015$
$= 1.05114$ cao A1 3

For Substituting **their** (0.01) into their (a) result
[0.1, 0.001, 0.25, 0.025,0.0025 acceptable but **not** 0.005 or 1.005]
First A1 (f.t.): Substitution of (**0.01**) into **their 4 termed** expression in (a)
Answer with no working scores **no marks** (calculator gives this answer)

[7]

7. (a) $1 + 6kx$ [Allow unsimplified versions, e.g. $1^6 + 6(1^5)kx$, ${}^6C_0 + {}^6C_1kx$] B1

$+\frac{6\times5}{2}(kx)^2+\frac{6\times5\times4}{3\times2}(kx)^3$ [See below for acceptable versions] M1A1 3

N.B. THIS NEED NOT BE SIMPLIFIED FOR THE A1 (isw is applied)

The terms can be 'listed' rather than added.

Requires correct structure: 'binomial coefficients' (perhaps from Pascal's triangle), increasing powers of x.
Allow a 'slip' or 'slips' such as:

$=\frac{6\times5}{2}kx^2+\frac{6\times5\times4}{3\times2}kx^3, +\frac{6\times5}{2}(kx)^2+\frac{6\times5}{3\times2}(kx)^3$

$+\frac{5\times4}{2}kx^2+\frac{5\times4\times3}{3\times2}kx^3, +\frac{6\times5}{2}x^2+\frac{6\times5\times4}{3\times2}x^3$

<u>But</u>: $15 + k^2x^2 + 20 + k^3x^3$ or similar is M0.

Both x^2 and x^3 terms must be seen.
$\begin{pmatrix}6\\2\end{pmatrix}$ and $\begin{pmatrix}6\\3\end{pmatrix}$ or equivalent such as ${}^6C_2$ and ${}^6C_3$ are acceptable, and even $\left(\frac{6}{2}\right)$ and $\left(\frac{6}{3}\right)$ are acceptable for the method mark.

A1: Any correct (possibly unsimplified) version of these 2 terms.
$\begin{pmatrix}6\\2\end{pmatrix}$ and $\begin{pmatrix}6\\3\end{pmatrix}$ or equivalent such as ${}^6C_2$ and ${}^6C_3$ are acceptable.

Descending powers of x:
Can score the M mark if the required first 4 terms are not seen.

Multiplying out $(1 + kx)(1 + kx)(1 + kx)(1 + kx)(1 + kx)(1 + kx)$:
A full attempt to multiply out (power 6)
B1 and A1 as on the main scheme.

(b) $6k = 15k^2$ $k = \frac{2}{5}$ (or equiv. fraction, or 0.4) (Ignore $k = 0$, if seen) M1A1cso 2

M: Equating the coefficients of x and x^2 (even if trivial, e.g. $6k = 15k$).
Allow this mark also for the 'misread': equating the coefficients of x^2 and x^3
An equation is k alone is required for this M mark, although...
... condone $6kx = 15k^2x^2 \Rightarrow (6k = 15k^2 \Rightarrow)\ k = \frac{2}{5}$.

(c) $c = \frac{6\times5\times4}{3\times2}\left(\frac{2}{5}\right)^3 = \frac{32}{25}$ (or equiv. fraction, or 1.28) A1cso 1

(Ignore x^3, so $\frac{32}{25}x^3$ is fine)

[6]

8. (a) $(1 - 2x)^5 = 1 + 5 \times (-2x) + \frac{5\times4}{2!}(-2x)^2 + \frac{5\times4\times3}{3!}(-2x)^3 + \ldots$ B1, A1,

$= 1 - 10x + 40x^2 - 80x^3 + \ldots$ A1 4

$1 - 10x$
$1 - 10x$ must be seen in this simplified form in (a). B1

Correct structure: 'binomial coefficients' (perhaps from Pascal's triangle), increasing powers of x.
Allow slips.

Accept other forms: ${}^5C_1$, $\begin{pmatrix}5\\1\end{pmatrix}$, also condone $\left(\frac{5}{1}\right)$ but must be attempting to use 5.
Condone use of invisible brackets and using $2x$ instead of $-2x$.
Powers of x: at least 2 powers of the type $(2x)^a$ or $2x^a$ seen for $a \geq 1$.

$40x^2$ (1st A1) A1

$-80x^3$ (2nd A1) A1

Allow commas between terms. Terms may be listed rather than added

Allow 'recovery' from invisible brackets, so

$1^5 + \binom{5}{1}1^4.-2x + \binom{5}{2}1^3.-2x^2 + \binom{5}{3}1^2.-2x^3 = 1 - 10x + 40x^2 - 80x^3 + ...$

gains full marks.

$1 + 5 \times (2x) + \frac{5\times4}{2!}(2x)^2 + \frac{5\times4\times3}{3!}(2x)^3 + ... = 1 + 10x + 40x^2 + 80x^3 + ...$

gains B0M1A1A0

Misread: first 4 terms, descending terms: if correct, would score B0, 1st A1: one of $40x^2$ and $-80x^3$ correct; 2nd A1: both $40x^2$ and $-80x^3$ correct.

Long multiplication

$(1-2x)^2 = 1 - 4x + 4x^2$, $(1-2x)^3 = 1 - 6x + 12x^2 - 8x^3$,

$(1-2x)^4 = 1 - 8x + 24x^2 - 32x^3$ $\{+ 16x^4\}$

$(1-2x)^5 = 1 - 10x + 40x^2 + 80x^3 + \ldots$

$1 - 10x$

$1 - 10x$ must be seen in this simplified form in (a). B1

Attempt repeated multiplication up to and including $(1-2x)^5$

$40x^2$ (1st A1) A1

$-80x^3$ (2nd A1) A1

Misread: first 4 terms, descending terms: if correct, would score B0, 1st A1: one of $40x^2$ and $-80x^3$ correct;
2nd A1: both $40x^2$ and $-80x^3$ correct.

(b) $(1+x)(1-2x)^5 = (1+x)(1-10x+\ldots)$

$= 1 + x - 10x + \ldots$

$\approx 1 - 9x$ (*) A1 2

Use their (a) and attempt to multiply out; terms (whether correct or incorrect) in x^2 or higher can be ignored.

If their (a) is correct an attempt to multiply out can be implied from the correct answer, so $(1+x)(1-10x) = 1 - 9x$ will gain A1.

If their (a) is correct, the 2nd bracket must contain at least $(1 - 10x)$ and an attempt to multiply out for the M mark. An attempt to multiply out is an attempt at 2 out of the 3 relevant terms (N.B. the 2 terms in x^1 may be combined – but this will still count as 2 terms).

If their (a) is incorrect their 2nd bracket must contain all the terms in x^0 and x^1 from their (a)

AND an attempt to multiply all terms that produce terms in x^0 and x^1.

N.B. $(1+x)(1-2x)^5 = (1+x)(1-2x)$ [where $1 - 2x + \ldots$ is NOT

the candidate's answer to (a)]
$= 1 - x$
i.e. candidate has ignored the power of 5: M0
N.B. The candidate may start again with the binomial expansion for
$(1 - 2x)^5$ in (b). If correct (only needs $1 - 10x$) may gain A1
even if candidate did not gain B1 in part (a).

N.B. Answer given in question. A1

Example

Answer in (a) is $= 1+10x + 40x^2 - 80x^3 +...$

$(1 + x)(1 + 10x) = 1 + 10x + x$
$= 1 + 11x$ A0

[6]

9. $(2 + x)^6 = 64 \ldots$ B1

$(6 \times 2^5 \times x) + \left(\dfrac{6 \times 5}{2} \times 2^4 \times x^2\right)$, $+192x$, $+240x^2$ A1,A1 4

The terms can be 'listed' rather than added.

Requires correct structure: 'binomial coefficients' (perhaps from Pascal's triangle), increasing powers of one term, decreasing powers of the other term (this may be 1 if factor 2 has been taken out). Allow 'slips'.

$\binom{6}{1}$ *and* $\binom{6}{2}$ *or equivalent are acceptable,*

or even $\left(\dfrac{6}{1}\right)$ *and* $\left(\dfrac{6}{2}\right)$.

Decreasing powers of x:

Can score only the M mark.

64(1 +……), even if all terms in the bracket are correct, scores max. B1M1A0A0.

[4

10. (a) $(1 + 6x)^4 = 1 + 4(6x) + 6(6x)^2 + 4(6x)^3 + (6x)^4$ A1
$= 1 + 24x + 216x^2 + 864x^3 + 1296x^4$ A1 3

(b) substitute $x = 100$ to obtain
$601^4 = 1 + 2400 + 2160000 + 864000000 + 129600000000$ M1o.e.
$= 130{,}466{,}162{,}401$ A1 2

[5]

11. (a) $(1+px)^9 = 1 + 9px; \quad + \binom{9}{2}(px)^2$ B1 B1 2

(b) $9p = 36$, so $\underline{p = 4}$ A1

$q = \dfrac{9 \times 8}{2}p^2$ or $36p^2$ or $36p$ if that follows from their (a)

So $\underline{q = 576}$ A1cao 4

[6]

(a) 2^{nd} B1 for $\binom{9}{2}(px)^2$ or better. Condone "," not "+".

(b) 1^{st} for a linear equation for p.
2^{nd} for either printed expression, follow through their p.

N.B. $1 + 9px^2 + 36px^2$ leading to $p = 4$, $q = 144$ scores B1B0 A1M1A0 i.e 4/6

12. (a) $1 + 12px, +66p^2x^2$ B1, B1 2

accept any correct equivalent

(b) $12p = -q$, $66p^2 = 11q$

Forming 2 equations by comparing coefficients

Solving for p or q
$p = -2$, $q = 24$ A1A1 4

[6]

13. (a) Writes down binomial expansion up to and including term in x^3,

allow ${}^nC_r$ notation $1 + nax + n(n-1)\dfrac{a^2x^2}{2} + \dfrac{n(n-1)(n-2)}{6}a^3x^3$

(condone errors in powers of a)

States $na = 15$ B1

Puts $\dfrac{n(n-1)a^2}{2} = \dfrac{n(n-1)(n-2)a^3}{6}$ dM1

(condone errors in powers of a)

$3 = (n-2)a$

Solves simultaneous equations in n and a to obtain
$a = 6$, and $n = 2.5$ A1 A1 6

[n.b. Just writes a = 6, and n = 2.5 following no working or following errors allow the last A1 A1]

(b) Coefficient of $x^3 = 2.5 \times 1.5 \times 0.5 \times 6^3 \div 6 = 67.5$ B1 1

(or equals coefficient of $x^2 = 2.5 \times 1.5 \times 6^2 \div 2 = 67.5$)

[7]

14. $(3 + 2x)^5 = (3^5) + \binom{5}{1}3^4 \, . \, (2x) + \binom{5}{2}3^3 \, (2x)^2 + \ldots$

$= \underline{243, +810x, +1080x^2}$ B1, A1, A1 4

[4]

15. (a) $\left(k + \frac{x}{2}\right)^5 = k^5 + 5k^4 \frac{x}{2} + 10k^3\left(\frac{x}{2}\right)^2 + 10k^2\left(\frac{x}{2}\right)^3 + \ldots$ A1 2

Need not be simplified. Accept $^5C_1$, $^5C_2$, $^5C_3$

(b) $10k^3\left(\frac{x}{2}\right)^2 = 540x^2$ with or without x^2

Leading to $k = 6$ (*) cso A1 2

Or substituting $k = 6$ into $^5C_2k^3\left(\frac{x}{2}\right)^2$ and simplifying to $540x^2$.

(c) Coefficient of x^3 is $10 \times 6^2 \times \frac{1}{2^3} = 45$ A1 2

[6]

16. (a) $(x^3)^{12}; \ldots + \binom{12}{1}(x^3)^{11}\left((-)\frac{1}{2x}\right) + \binom{12}{2}(x^3)^{10}\left((-)\frac{1}{2x}\right)^2 + \ldots$ B1;

[For needs binomial coefficients, ^nC_r form OK at least as far as shown]

Correct values for $^nC_r s$: 12, 66, 220 used (may be implied) B1

$\{(x^3)^{12} + 12(x^3)^{11}\left(-\frac{1}{2x}\right) + 66(x^3)^{10}\left(-\frac{1}{2x}\right)^2 + 220(x^3)^9\left(-\frac{1}{2x}\right)^3 \ldots$

$= x^{36} - 6x^{32} + \frac{33}{2}x^{28} - \frac{55}{2}x^{24}$ A2(1,0) 5

(b) Term involving $(x^3)^3\left((-)\frac{1}{2x}\right)^9$;

$\text{coeff} = \frac{12.11.10}{3.2.1}\left((-)\frac{1}{2}\right)^9$ A1

$= -\frac{55}{128}$ (or –0.4296875) A1 3

[8]

17. (a) $1 + nax, + \frac{n(n-1)}{2}(ax)^2 + \frac{n(n-1)(n-2)}{6}(ax)^3 + \ldots$ B1,B1 2

accept 2!, 3!

(b) $na = 8,\ \frac{n(n-1)}{2}a^2 = 30$

both

$\frac{n(n-1)}{2}.\frac{64}{n^2} = 30,\ \frac{\frac{8}{a}\left(\frac{8}{a}-1\right)a^2}{2} = 30$

either

$n = 16,\ a = \frac{1}{2}$ A1, A1 4

(c) $\frac{16.15.14}{6}.\left(\frac{1}{2}\right)^3 = 70$ A1 2

[8]

18. $(2 - px)^6 = 2^6 + \binom{6}{1}2^5(-px) + \binom{6}{2}2^4(-px)^2$ $\binom{n}{r}$ okay

Coeff. of x or x^2

$= 64 + 6 \times 2^5(-px);\ \ + 15 \times 2^4(-px)^2$ A1; A1

No $\binom{n}{r}$

$15 \times 16p^2 = 135$ $\Rightarrow$ $p^2 = \frac{9}{16}$ or $\underline{p = \frac{3}{4}}$ (only) A1

$-6.32p = A$ $\Rightarrow$ $\underline{A = -144}$ A1 ft (their p (> 0)) 7

Condone lost or extra '–' signs for M marks but A marks must be correct.
Final A1 ft is for –192x (their p > 0)

[7]

19. $(1 + px)^n \equiv 1 + npx, + \frac{n(n-1)p^2x^2}{2} + \ldots$ B1, B1

Comparing coefficients: $np = -18,\ \frac{n(n-1)}{2} = 36$ A1

Solving $n(n - 1) = 72$ to give $n = 9$; $p = -2$ A1; A1 ft

[7]

20. (a) $\left(2+\frac{1}{4}x\right)^9 = 2^9 + 9 \times 2^8\left(\frac{1}{4}x\right) + \frac{9\times 8}{2}(2^7)\frac{x^2}{16} + \frac{9\times 8\times 7}{6} \times 2^6 \times \frac{x^3}{64}$ M1 B1

(for descending powers of 2 and ascending powers of x; B1 for coefficients 1, 9, 36, 84 in any form, as above)

$= 512 + 576x, + 288x^2 + 84x^3$ A1, A1 4

(b) $x = \frac{1}{10}$ gives

$(2.025)^9 = 512 + 57.6 + 2.88 + 0.084$

$= 572.564$ A1 2

[6]

21. (a) $\frac{n(n-1)}{2!}k^2 = \frac{n(n-1)(n-2)}{3!}k^3$

One coefficient (no $\binom{n}{r}$*)*

A correct equation, no cancelling A1

e.g. $3k^2 = (n-2)k^3$

Cancel at least n (n – 1)

$3 = (n-2)k$ **(*)** A1 cso 4

(b) $A = nk = 4$ B1

$3 = 4 - 2k$

So $k = \frac{1}{2}$, and $n = 8$ A1, A1 4

[8]

1. In part (a), most candidates exhibited understanding of the structure of a binomial expansion and were able to gain at least the method mark. Coefficients were generally found using the $(1 + x)^n$ binomial expansion formula, but Pascal's triangle was also popular. The correctly simplified third and fourth terms, $21a^2x^2$ and $35a^3x^3$, were often obtained and it was pleasing that $21ax^2$ and $35ax^3$ appeared less frequently than might have been expected from the evidence of previous papers. Candidates tend to penalise themselves due to their reluctance to use brackets in terms such as $21(ax)^2$ and $35(ax)^3$.

Part (b) was often completed successfully, but some candidates included powers of x in their 'coefficients'. There is still an apparent lack of understanding of the difference between 'coefficients' and 'terms'. Although the question asked for the 'values' of a, some candidates gave only $a = 5$, ignoring the other possibility $a = -5$.

2. This binomial expansion was answered well, with a majority of the candidates scoring three or four marks. The binomial coefficients were usually correct, though a few used ${}^5C_r$ instead of ${}^6C_r$ .Those using the $(a + b)^n$ formula were the most accurate. The majority of errors with that method being with +/- signs: using x instead of $-x$, $(-x)^2$ becoming $-x^2$, not simplifying $1458(-x)$ to $-1458x$ or leaving as $+(-1458x)$. Attempts to take out the 3 to use the $(1 + x)^n$ expansion were generally less successful with candidates not raising 3 to a power or not dividing the x term by 3. There were a number of marks lost by slips such as miscopying 729 as 792 or 726, or neglecting the x in the second term.

3. In part (a), most candidates were aware of the structure of a binomial expansion and were able to gain the method mark. Those who used the $(a + b)^n$ formula were usually able to pick up accuracy marks but many of those who attempted to use the $(1 + x)^n$ version made mistakes in simplifying terms, often taking out 2 as a factor rather than 2^7. Coefficients were generally found using ${}^nC_r$, but Pascal's triangle was also frequently seen. The simplified third term was often given as $672kx^2$ instead of $672k^2 x^2$, but this mistake was much less common than in similar questions on previous papers.

Part (b) was often completed successfully, but some candidates included powers of x in their 'coefficients'. Compared with recent papers, there seems to be some improvement in the understanding of the difference between 'coefficients' and 'terms'. Accuracy mistakes, including multiplying the wrong coefficient by 6, were common.

4. This binomial expansion was answered well with a majority of the candidates scoring full marks. The most common errors involved signs and slips in evaluating the powers and binomial coefficients. A number of weaker candidates changed the question and instead expanded $(1 \pm 2x)^5$. This gained no credit.

5. In part (a), most candidates were aware of the structure of a binomial expansion and were able to gain the method mark. Coefficients were generally found using ${}^nC_r$, but Pascal's triangle was also frequently seen. The most common mistake was to omit the powers of a, either completely or perhaps in just the simplified version of the answer.
Part (b) was often completed successfully, but a significant number of candidates included powers of x in their 'coefficients', resulting in some very confused algebra and indicating misunderstanding of the difference between 'coefficients' and 'terms'. Sometimes the wrong

coefficient was doubled and sometimes the coefficients were equated with no doubling. Some candidates, having lost marks in part (a) due to the omission of powers of a, recovered in part (b) and achieved the correct answer.

6. It was pleasing to see that most candidates could make some headway in part (a) and many candidates gained full marks. The usual errors of omitting brackets around $\frac{x}{2}$, and using $\left(\frac{10}{r}\right)$ for $\binom{10}{r}$, were seen, but not as frequently as on previous occasions. It was also common to see the coefficients of powers of x not reduced to their simplest form. Solutions to part (b) were variable, with many candidates not able to find the appropriate value of x to use; frequently 0.005 was substituted into a correct, or near correct expansion expression found in (a). Just writing the answer down, with no working at all, gained no marks.

7. This question was not particularly well answered, many candidates having difficulty coping with the constant k in their binomial expansion. Pascal's Triangle was sometimes used (rather than the binomial expansion formula) in part (a), and while terms did not need to be simplified at this stage, mistakes in simplification frequently spoilt solutions to parts (b) and (c). A very common mistake was to have kx^2 and kx^3 rather than $(kx)^2$ and $(kx)^3$. Candidates who made this mistake often produced $6k = 15k$ in part (b) and were then confused (but often proceeded to obtain non-zero solutions of this equation). The difference between 'coefficients' and 'terms' was not well understood, so $6kx = 15k^2x^2$ was often seen. Sometimes 'recovery' led to the correct answers in parts (b) and (c), but sometimes tried to solve an equation in two unknowns and made no progress.

8. Part (a) was usually answered well with many candidates showing understanding of the structure of a binomial expansion. Common errors included the use of x or $2x$ instead of $–2x$ and the careless use of brackets. Some candidates did not spot the relationship between parts (a) and (b) and started again with the expansion of $(1 – 2x)^5$, others used the whole of their answer to part (a) when they only need to use $1 – 10x$. A number of candidates substituted a value for x and then attempted show that their expressions were approximately equal and there were also a few who tried to fool the examiners by writing the given answer after several lines of incorrect working.

9. This question was well answered by many candidates. However, those who attempted to take out a factor of 2 before expanding often failed to realise that they needed then to multiply by 2^6. Sometimes the '2' was completely ignored and the expansion of $(1 +x)^6$ was given. In general, candidates fared better if they used a formula for $(a + b)$ rather than $(1 + x)^n$. Some candidates failed to simplify their terms or made careless mistakes in their attempts to simplify.

10. Pure Mathematics P2

Candidates started this question confidently. The overwhelming majority of them completed the binomial expansion correctly. A few only went as far as the term in x^3, and others suggested that the expansion might continue beyond the term in x^4 by giving a set of dots on the end of their answer. Only a handful ignored the fact that the expansion needed to involve powers of $6x$ and/or the coefficients 1, 4, 6, 4 and 1.

Many candidates were familiar with the method for (b) and correctly identified the need to substitute 100 in place of x in their expansion. It was disappointing that so many candidates then went on to add the numbers using their calculators and did not appreciate that they should have expected their answer to end with the digit 1. Some candidates simply offered a calculator value for 601^4 with no evidence of use of their expansion at all. Many answers were expressed in standard form, often with the value rounded to 3 or 4 significant figures.

Core Mathematics

This question was well done and many scored full marks. In part (a), the great majority of candidates translated the graph in the correct direction. A substantial minority of candidates interchanged parts (b) and (c), giving the graph for part (b) as their answer to part (c) and the graph for part (c) as their answer to part (b). In part (c), it was not uncommon for candidates to have no branch of the curve for $x < 0$ and two branches of the curve with $x > 0$. The coordinates of the maximum points were usually correctly given.

11. This question was answered well by the majority of the candidates. The most common error was to fail to square the p in the third term giving $36px^2$ which led to a value of 144 for q. Most candidates were able to use their binomial expansion in part (a) to form two equations in part (b) for p and q. Occasionally problems were caused by candidates including an extra x term but in most cases success in part (a) was followed by a sensible attempt at part (b). A few candidates used Pascal's triangle to evaluate their coefficients in part (a) but there were often errors in the triangle, this is not an approach the examiners would recommend.

12. Pure Mathematics P2

Many candidates scored very well on this question.

(a) The coefficients of 12 and 66 were almost always correct, although 66p was a popular alternative to $66p^2$. A few candidates were confused between $\binom{12}{n}$ and $\frac{12}{n}$. Some left out the first term, and some had correct coefficients matched with the wrong powers of x.

(b) There was some confusion between coefficients and terms, and also several candidates who tried to substitute $-q$ and $11q$ for x and x^2 respectively. When comparing coefficients some students got into problems by including x's. Many however did obtain the correct pair of simultaneous equations and were largely successful at solving them to find p and q. There were some sign errors in the solutions, particularly when squaring $\left(\frac{-q}{12}\right)$.

Core Mathematics

Most candidates had a good idea of how to use a binomial expansion, but some had difficulty with simplification, perhaps leaving their terms in forms such as $\begin{pmatrix}12\\1\end{pmatrix}1^{11}(px)$ and not knowing how to proceed. The most common mistake in part (a) was the failure to square p in the coefficient of the x^2 term. Those who made this mistake were still able to score method marks in part (b), but then made no further progress because their equations for p and q yielded only zero roots.

Many candidates had difficulty with part (b). Often there was confusion between 'coefficient' and 'term', and statements such as $12px = -q$ and $66p^2x^2 = 11q$ led to attempts to solve two equations in three unknowns. Rather than comparing terms, some candidates tried to write down a single expansion with terms involving both p and q, and others used $-q$ and $11q$ as values of x in their expansion.

13. Generally most candidates knew the binomial expansion, with a few not including the correct powers of a. There was clear evidence of inefficient cancelling, and there were many long and inelegant methods, so that many candidates were unable to obtain values of a and n from the complicated cubic equation which they had found. Early simplification gave rise to a simple linear equation.

14. This was usually answered very well with many totally correct solutions. Candidates who tried to remove a factor of 3^5, often made mistakes or forgot to multiply their terms by 243. By far the most common error was a failure to include brackets in the 3rd term and $2x^2$ rather than $(2x)^2$ was seen all too often. A few candidates had not understood the $\begin{pmatrix}n\\r\end{pmatrix}$ notation for binomial coefficients. Some wrote $\left(\dfrac{5}{2}\right)$ and occasionally this was interpreted as 2.5.

15. The majority of candidates knew what was required in this question and many gained full marks. However there was much careless work with brackets and the error of having k^4 in the term in x^3 was not infrequently seen. Candidates who began by writing the expression as $k^5\left(1+\dfrac{x}{2k}\right)^5$ were rarely successful. Candidates who completed part (a) and simplified their expressions nearly always completed the question correctly.

16. Most candidates seemed to have a knowledge of the binomial expansion, with few omitting the binomial coefficients, and many correct answers to part (a) were seen. However, there were many different types of error seen, mainly due to lack of care in manipulation. The fact that both terms in the bracket were functions of x proved a problem and candidates who felt they had to take out the factor of x^3 or $\left(-\dfrac{1}{2x}\right)$ to ensure that one term in the bracket was 1 often made mistakes; it was quite common to see $(x^3)^{12}$ become x^{15}, or $\left(-\dfrac{1}{2x}\right)^{12}$ become $\dfrac{1}{2x^{12}}$, for example. Sign errors were common and expansions in ascending powers of x were frequently seen.. Many candidates did not offer an attempt at part (b) and others showed that they did not understand what was expected. Those that knew what to do often lost one mark for omission of the minus sign. Candidates who had expanded, in ascending powers of x in part (a), often had the answer as their fourth term. Some did not realise that, but if they did then all marks for this part were available.

17. In part (a) the majority of candidates obtained at least one mark although the powers of a were often incorrect. In this part of the question expressions ${}_nC_2$ and $\binom{n}{2}$ were accepted for full marks but in part (b) it was necessary for the candidate to show that they knew what these expressions meant to gain marks and, despite the fact that definitions of the binomial coefficients are given in the formula booklet, many were unable to do this. Those who did obtain two algebraic equations often found the resulting algebra very testing and even those who did complete correctly often needed two or three attempts at solving the simultaneous equations with a subsequent loss of time. Those who were successful with part (a) and (b) usually completed (c) correctly.

18. The candidates showed a good understanding of the binomial expansion and there were plenty of fully correct solutions. There was though, even amongst some better candidates, a general lack of attention to accuracy often caused by poor (or completely absent) use of brackets. The negative sign on the coefficient of x was often lost and $(px)^2$ often became px^2 as work continued. A number of candidates were not sure how to deal with the 2. Some completely ignored it and others tried to deal with it but gave expressions such as

$2^6(1-px)^6$ or $2(1-p\frac{x}{2})^6$. Most used the $\binom{n}{r}$ formulae for the binomial coefficients but a small minority tried to use Pascal's triangle, a method we would not recommend.

19. Although many good solutions, set out as on the mark scheme, were seen there were many disappointing attempts, often showing a lack of understanding of the notation used. Solutions were often disorganised, showing no real strategy, and too many candidates just tried to fit an n and a p to the data.

Although the expansion $(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \ldots$ ($|x| < 1$, n e R) given in the Formulae Booklet, is essentially for questions on P3, it is a pity that many candidates seemed unaware that they had $\binom{n}{1}$ and $\binom{n}{2}$ written out in terms of n, so that the first three terms in the expansion of $(1+px)^n$ could have been written down immediately as

$1 + n(px) + \frac{n(n-1)}{2}(px)^2$.

A common slip was to state that $np = +18$, and statements like $n = -18$ and $\frac{n(n-1)}{2}p^2 = 36$ were frequently seen. Too many candidates did not recognise $\frac{n(n-1)}{2} = 36$ as a quadratic equation in n, and, strangely, some substituted $n = \frac{-18}{p}$ into this equation and correctly solved the resulting quadratic in p.

20. No Report available for this question.

21. No Report available for this question.